Couplings between a collection of BF models and a set of three-form gauge fields

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Abstract

Consistent interactions that can be added to a free, Abelian gauge theory comprising a collection of BF models and a set of three-form gauge fields are constructed from the deformation of the solution to the master equation based on specific cohomological techniques. Under the hypotheses of smooth, local, PT invariant, Lorentz covariant, and Poincaré invariant interactions, supplemented with the requirement on the preservation of the number of derivatives on each field with respect to the free theory, we obtain that the deformation procedure modifies the Lagrangian action, the gauge transformations as well as the accompanying algebra.

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Topological field theories [1]–[2] are important in view of the fact that certain interacting, non-Abelian versions are related to a Poisson structure algebra [3] present in various versions of Poisson sigma models [4]–[10], which are known to be useful at the study of two-dimensional gravity [11]–[20] (for a detailed approach, see [21]). It is well known that pure three-dimensional gravity is just a BF theory. Moreover, in higher dimensions general relativity and supergravity in Ashtekar formalism may also be formulated as topological BF theories with some extra constraints [22]–[25]. In view of

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these results, it is important to know the self-interactions in BF theories as well as the couplings between BF models and other theories. This problem has been considered in literature in relation with self-interactions in various classes of BF models [26]–[32] and couplings to matter fields [33] and vector fields [34]–[35] by using the powerful BRST cohomological reformulation of the problem of constructing consistent interactions within the Lagrangian [36] or the Hamiltonian [37] setting. Other aspects concerning interacting, topological BF models can be found in [38]–[40]. On the other hand, models with p-form gauge fields play an important role in string and superstring theory as well as in supergravity. In particular, three-form gauge fields are important due to their presence in eleven-dimensional supergravity. Based on these considerations, the study of interactions between BF models and three-forms appears as a topic that might enlighten certain aspects in both gravity and supergravity theories.

The scope of this paper is to investigate the consistent interactions that can be added to a free, Abelian gauge theory consisting of a collection of BF models and a set of three-form gauge fields. This matter is addressed by means of the deformation of the solution to the master equation from the BRST-antifield formalism [36]. Under the hypotheses of smooth, local, PT invariant, Lorentz covariant, and Poincaré invariant interactions, supplemented with the requirement on the preservation of the number of derivatives on each field with respect to the free theory, we obtain the most general form of the theory that describes the cross-couplings between a collection of BF models and a set of three-form gauge fields. The resulting interacting model is accurately formulated in terms of a gauge theory with gauge transformations that close according to an open algebra (the commutators among the deformed gauge transformations only close on the stationary surface of deformed field equations), which are on-shell, second-order reducible.

Our starting point is a four-dimensional, free theory, describing a collection of topological BF models (each of them involving two types of one-forms, a set of scalar fields, and a system of two-forms) and a set of Abelian 3-form gauge fields, with the Lagrangian action

$$S_0\left[A_\mu^a, H_\mu^a, \varphi_a, B_a^{\mu\nu}, V_{\mu\nu\rho}^A\right] = \int d^4x \left(H_\mu^a \partial^\mu \varphi_a + \tfrac{1}{2} B_a^{\mu\nu} \partial_{[\mu} A_{\nu]}^a - \tfrac{1}{2\cdot 4!} F_{\mu\nu\rho\lambda}^A F_A^{\mu\nu\rho\lambda}\right). \tag{1}$$

The collection indices from the three-form sector (capital, Latin letters) are lowered with the (non-degenerate) metric k_{AB} induced by the Lagrangian

density $F^{A}_{\mu\nu\rho\lambda}F^{\mu\nu\rho\lambda}_{A}$ in (1) (i.e. $F^{\mu\nu\rho\lambda}_{A}=k_{AB}F^{B\mu\nu\rho\lambda}$) and are raised with its inverse, of elements k^{AB}. The field strength of a given three-form gauge field $V^{A}_{\mu\nu\rho}$ is defined in the standard manner as $F^{A}_{\mu\nu\rho\lambda}=\partial_{[\mu}V^{A}_{\nu\rho\lambda]}$. Everywhere in this paper the notation $[\mu\ldots\lambda]$ signifies complete antisymmetry with respect to the (Lorentz) indices between brackets, with the conventions that the minimum number of terms is always used and the result is never divided by the number of terms. The above action is invariant under the gauge transformations

$$\delta_{\epsilon}A^{a}_{\mu}=\partial_{\mu}\epsilon^{a},\quad \delta_{\epsilon}\varphi_{a}=0, \tag{2}$$

$$\delta_{\epsilon}H^{a}_{\mu}=2\partial^{\nu}\epsilon^{a}_{\mu\nu},\quad \delta_{\epsilon}B^{\mu\nu}_{a}=-3\partial_{\rho}\epsilon^{\mu\nu\rho}_{a},\quad \delta_{\epsilon}V^{A}_{\mu\nu\rho}=\partial_{[\mu}\epsilon^{A}_{\nu\rho]}, \tag{3}$$

where all the gauge parameters ϵ^{a}, $\epsilon^{a}_{\mu\nu}$, $\epsilon^{\mu\nu\rho}_{a}$ and $\epsilon^{A}_{\mu\nu}$ are bosonic, with the last three sets completely antisymmetric. The gauge algebra associated with (2) and (3) is Abelian.

We observe that if in (3) we make the transformations $\epsilon^{a}_{\mu\nu}\rightarrow\epsilon^{a}_{\mu\nu}(\theta)=-3\partial^{\rho}\theta^{a}_{\mu\nu\rho}$, $\epsilon^{\mu\nu\rho}_{a}\rightarrow\epsilon^{\mu\nu\rho}_{a}(\theta)=4\partial_{\lambda}\theta^{\mu\nu\rho\lambda}_{a}$, $\epsilon^{A}_{\mu\nu}\rightarrow\epsilon^{A}_{\mu\nu}(\theta)=\partial_{[\mu}\theta^{A}_{\nu]}$, then the gauge variations from (3) identically vanish $\delta_{\epsilon(\theta)}H^{a}_{\mu}\equiv0$, $\delta_{\epsilon(\theta)}B^{\mu\nu}_{a}\equiv0$, $\delta_{\epsilon(\theta)}V^{A}_{\mu\nu\rho}\equiv0$. Moreover, if we perform the changes $\theta^{a}_{\mu\nu\rho}\rightarrow\theta^{a}_{\mu\nu\rho}(\phi)=4\partial^{\lambda}\phi^{a}_{\mu\nu\rho\lambda}$, $\theta^{A}_{\mu}\rightarrow\theta^{A}_{\mu}(\phi)=\partial_{\mu}\phi^{A}$, with $\phi^{a}_{\mu\nu\rho\lambda}$ completely antisymmetric functions and ϕ an arbitrary scalar field, then the transformed gauge parameters identically vanish $\epsilon^{a}_{\mu\nu}(\theta(\phi))\equiv0$, $\epsilon^{A}_{\mu\nu}(\theta(\phi))\equiv0$. Meanwhile, there is no non-vanishing, local transformation of $\phi^{a}_{\mu\nu\rho\lambda}$ and ϕ^{A} that annihilates $\theta^{a}_{\mu\nu\rho}(\phi)$ and respectively $\theta^{A}_{\mu}(\phi)$, and hence no further local reducibility identity. All these allow us to conclude that the generating set of gauge transformations (2) and (3) is off-shell, second-order reducible.

The construction of the BRST symmetry for this free theory debuts with the identification of the algebra on which the BRST differential s acts. The generators of the BRST algebra are of two kinds: fields/ghosts and antifields. The ghost spectrum for the model under study comprises the fermionic ghosts $\eta^{\alpha_1}=\left(\eta^{a},C^{a}_{\mu\nu},\eta^{\mu\nu\rho}_{a},\bar{\eta}^{A}_{\mu\nu}\right)$ associated with the gauge parameters $\left(\epsilon^{a},\epsilon^{a}_{\mu\nu},\epsilon^{\mu\nu\rho}_{a},\epsilon^{A}_{\mu\nu}\right)$ from (2) and (3), the bosonic ghosts for ghosts $\eta^{\alpha_2}=\left(C^{a}_{\mu\nu\rho},\eta^{\mu\nu\rho\lambda}_{a},\bar{\eta}^{A}_{\mu}\right)$ due to the first-order reducibility parameters $\left(\theta^{a}_{\mu\nu\rho},\theta^{\mu\nu\rho\lambda}_{a},\theta^{A}_{\mu}\right)$, and also the fermionic ghost for ghosts for ghosts $\eta^{\alpha_3}=\left(C^{a}_{\mu\nu\rho\lambda},\bar{\eta}^{A}\right)$ corresponding to the second-order reducibility parameters $\left(\phi^{a}_{\mu\nu\rho\lambda},\phi^{A}\right)$. The antifield spectrum is organized into the antifields $\Phi^{*}_{\alpha_0}=\left(A^{*\mu}_{a},H^{*\mu}_{a},\varphi^{*a},B^{*a}_{\mu\nu},V^{*\mu\nu\rho}_{A}\right)$ of the original tensor fields and those corresponding to the ghosts, denoted by $\eta^{*}_{\alpha_1}=\left(\eta^{*}_{a},C^{*\mu\nu}_{a},\eta^{*a}_{\mu\nu\rho},\bar{\eta}^{*\mu\nu}_{A}\right)$, $\eta^{*}_{\alpha_2}=\left(C^{*\mu\nu\rho}_{a},\eta^{*a}_{\mu\nu\rho\lambda},\bar{\eta}^{*\mu}_{A}\right)$, and respectively by

$\eta^*_{\alpha_3} = \left(C^{*\mu\nu\rho\lambda}_a, \bar{\eta}^*_A\right)$. The Grassmann parity of a given antifield is opposite to that of the associated field/ghost.

The BRST symmetry of this free theory simply decomposes as the sum between the Koszul-Tate differential δ and the exterior derivative along the gauge orbits γ, $s = \delta + \gamma$, where the degree of δ is the antighost number ($\mathrm{antigh}\,(\delta) = -1$, $\mathrm{antigh}\,(\gamma) = 0$), and that of γ is the pure ghost number ($\mathrm{pgh}\,(\gamma) = 1$, $\mathrm{pgh}\,(\delta) = 0$). The grading of the BRST differential is named ghost number (gh) and is defined in the usual manner like the difference between the pure ghost number and the antighost number, such that $\mathrm{gh}\,(\delta) = \mathrm{gh}\,(\gamma) = \mathrm{gh}\,(s) = 1$. According to the standard rules of the BRST method, the corresponding degrees of the generators from the BRST complex are valued like: $\mathrm{pgh}\,(\Phi^{\alpha_0}) = 0$, $\mathrm{pgh}\,(\eta^{\alpha_1}) = 1$, $\mathrm{pgh}\,(\eta^{\alpha_2}) = 2$, $\mathrm{pgh}\,(\eta^{\alpha_3}) = 3$, $\mathrm{pgh}\left(\Phi^*_{\alpha_0}\right) = \mathrm{pgh}\left(\eta^*_{\alpha_1}\right) = \mathrm{pgh}\left(\eta^*_{\alpha_2}\right) = \mathrm{pgh}\left(\eta^*_{\alpha_3}\right) = 0$, $\mathrm{agh}\,(\Phi^{\alpha_0}) = \mathrm{agh}\,(\eta^{\alpha_1}) = \mathrm{agh}\,(\eta^{\alpha_2}) = \mathrm{agh}\,(\eta^{\alpha_3}) = 0$, $\mathrm{agh}\left(\Phi^*_{\alpha_0}\right) = 1$, $\mathrm{agh}\left(\eta^*_{\alpha_1}\right) = 2$, $\mathrm{agh}\left(\eta^*_{\alpha_2}\right) = 3$, $\mathrm{agh}\left(\eta^*_{\alpha_3}\right) = 4$. The BRST differential is known to have a canonical action in a structure named antibracket and denoted by the symbol $(,)$ ($s\cdot = (\cdot, S)$), which is obtained by setting the fields/ghosts respectively conjugated to the corresponding antifields. The generator of the BRST symmetry is a bosonic functional of ghost number zero, which is solution to the classical master equation $(S, S) = 0$. The full solution to the master equation for the free model under study reads as

$$\begin{aligned} S &= S_0 + \int d^4x \left(A^{*\mu}_a \partial_\mu \eta^a + 2H^{*\mu}_a \partial^\nu C^a_{\mu\nu} - 3B^{*a}_{\mu\nu} \partial_\rho \eta^{\mu\nu\rho}_a \right. \\ &\quad + V^{*\mu\nu\rho}_A \partial_{[\mu} \bar{\eta}^A_{\nu\rho]} - 3C^{*\mu\nu}_a \partial^\rho C^a_{\mu\nu\rho} + 4\eta^{*a}_{\mu\nu\rho} \partial_\lambda \eta^{\mu\nu\rho\lambda}_a \\ &\quad \left. + \bar{\eta}^{*\mu\nu}_A \partial_{[\mu} \bar{\eta}^A_{\nu]} + 4C^*_{\mu\nu\rho} \partial_\lambda C^{\mu\nu\rho\lambda} + \bar{\eta}^{*\mu}_A \partial_\mu \bar{\eta}^A \right). \end{aligned} \tag{4}$$

Now, we consider the problem of constructing consistent interactions among the fields Φ^{α_0} such that the couplings preserve the field spectrum and the original number of gauge symmetries. The matter of constructing consistent interactions is addressed by means of reformulating this issue as a deformation problem of the solution to the master equation corresponding to the free theory [36]. Such a reformulation is possible due to the fact that the solution to the master equation contains all the information on the gauge structure of the theory. If an interacting gauge theory can be consistently constructed, then the solution S to the master equation associated with the free theory can be deformed into a solution $\bar{S}$

$$S \rightarrow \bar{S} = S + \lambda S_1 + \lambda^2 S_2 + \lambda^3 S_3 + \cdots, \tag{5}$$

of the master equation for the deformed theory

$$\left(\bar{S},\bar{S}\right)=0, \tag{6}$$

such that both the ghost and antifield spectra of the initial theory are preserved. Equation (6) splits, according to the various orders in λ, into

$$(S,S)=0, \tag{7}$$

$$2\left(S_1,S\right)=0, \tag{8}$$

$$2\left(S_2,S\right)+\left(S_1,S_1\right)=0, \tag{9}$$

$$\vdots$$

Equation (7) is fulfilled by hypothesis. The next one requires that the first-order deformation of the solution to the master equation, S_1, is a co-cycle of the "free" BRST differential. However, only cohomologically non-trivial solutions to (8) should be taken into account, as the BRST-exact ones (BRST co-boundaries) correspond to trivial interactions. This means that S_1 pertains to the ghost number zero cohomological space of s, $H^0(s)$, which is generically non-empty due to its isomorphism to the space of physical observables of the "free" theory. It has been shown (on behalf of the triviality of the antibracket map in the cohomology of the BRST differential) that there are no obstructions in finding solutions to the remaining equations, namely (9), etc.

The resolution of equations (8)–(9), etc., implies standard cohomological techniques related to the BRST differential of the free model under consideration. In the sequel we give the solutions to these equations without going into further details (to be reported elsewhere). The (non-trivial) solution to equation (8) can be shown to expand like $S_1=\int d^4x\left(\alpha_0+\alpha_1+\alpha_2+\alpha_3+\alpha_4\right)$, where $\mathrm{antigh}\left(\alpha_k\right)=k$. The component of antighost number four from the above decomposition reads as

$$\begin{aligned}\alpha_4 \;=\;& \left(P_{ab}\left(W\right)\right)^{\mu\nu\rho\lambda}\eta^a C^b_{\mu\nu\rho\lambda}-\tfrac{1}{4}\left(P^c_{ab}\left(M\right)\right)_{\mu\nu\rho\lambda}\eta^a\eta^b\eta_c^{\mu\nu\rho\lambda}\\ &+Q_{aA}\left(f\right)\eta^a\bar{\eta}^A+\tfrac{1}{4!}Q_{abcd}\left(f\right)\eta^a\eta^b\eta^c\eta^d+\tfrac{1}{2}Q^{ab}\left(f\right)\eta_{a\mu\nu\rho\lambda}\eta_b^{\mu\nu\rho\lambda},\end{aligned} \tag{10}$$

where we used the notations

$$\left(P_{\Delta}\left(\chi\right)\right)^{\mu\nu\rho\lambda}=\frac{\partial\chi_{\Delta}}{\partial\varphi_a}C_a^{*\mu\nu\rho\lambda}+\frac{\partial^2\chi_{\Delta}}{\partial\varphi_a\partial\varphi_b}\left(H_a^{*[\mu}C_b^{*\nu\rho\lambda]}+C_a^{*[\mu\nu}C_b^{*\rho\lambda]}\right)$$

$$
+\frac{\partial^3\chi_\Delta}{\partial\varphi_a\partial\varphi_b\partial\varphi_c}H_a^{*[\mu}H_b^{*\nu}C_c^{*\rho\lambda]}+\frac{\partial^4\chi_\Delta}{\partial\varphi_a\partial\varphi_b\partial\varphi_c\partial\varphi_d}H_a^{*\mu}H_b^{*\nu}H_c^{*\rho}H_d^{*\lambda}, \quad (11)
$$

$$
\begin{aligned}
Q_\Lambda(f) &= f_\Lambda^A\bar{\eta}_A^{*}-\left(P_\Lambda^A(f)\right)_\mu\bar{\eta}_A^{*\mu}-\left(P_\Lambda^A(f)\right)_{\mu\nu}\bar{\eta}_A^{*\mu\nu}\\
&+\left(P_\Lambda^A(f)\right)_{\mu\nu\rho}V_A^{*\mu\nu\rho}-\tfrac{1}{4!}\left(P_\Lambda^A(f)\right)_{\mu\nu\rho\lambda}F_A^{\mu\nu\rho\lambda}, \qquad (12)
\end{aligned}
$$

and

$$
(P_\Delta(\chi))^{\mu\nu\rho}=\frac{\partial\chi_\Delta}{\partial\varphi_a}C_a^{*\mu\nu\rho}+\frac{\partial^2\chi_\Delta}{\partial\varphi_a\partial\varphi_b}H_a^{*[\mu}C_b^{*\nu\rho]}+\frac{\partial^3\chi_\Delta}{\partial\varphi_a\partial\varphi_b\partial\varphi_c}H_a^{*\mu}H_b^{*\nu}H_c^{*\rho}, \qquad (13)
$$

$$
(P_\Delta(\chi))^{\mu\nu}=\frac{\partial\chi_\Delta}{\partial\varphi_a}C_a^{*\mu\nu}+\frac{\partial^2\chi_\Delta}{\partial\varphi_a\partial\varphi_b}H_a^{*\mu}H_b^{*\nu}, \quad (P_\Delta(\chi))^{\mu}=\frac{\partial\chi_\Delta}{\partial\varphi_a}H_a^{*\mu}. \quad (14)
$$

The functions $(P_{ab}(W))^{\mu\nu\rho\lambda}$ and $(P_{ab}^c(M))_{\mu\nu\rho\lambda}$ are obtained from (11) in which we replace χ_Δ with W_{ab} and respectively with M_{ab}^c, while the elements $Q_{aA}(f)$, $Q_{abcd}(f)$, and $Q^{ab}(f)$ result from the relations (12) and (11)–(14) where, instead of f_Λ^A, we put f_{aB}^A, f_{abcd}^A, and respectively f^{Aab}. (The objects $\left(P_\Lambda^A(f)\right)_{\mu\nu\rho\lambda}$, $\left(P_\Lambda^A(f)\right)_{\mu\nu\rho}$, $\left(P_\Lambda^A(f)\right)_{\mu\nu}$, and $\left(P_\Lambda^A(f)\right)_{\mu}$ are expressed by (11), (13), and (14) in which W_Δ is substituted by f_Λ^A). The quantities W_{ab}, M_{ab}^c, f_{aB}^A, f_{abcd}^A, and f^{Aab} are arbitrary functions of the undifferentiated scalar fields φ_a, with M_{ab}^c and f_{abcd}^A completely antisymmetric in their lower indices and f^{Aab} symmetric in its BF indices. The piece of antighost number three from S_1 is given by

$$
\begin{aligned}
\alpha_3 &= -(P_{ab}(W))^{\mu\nu\rho}\left(\eta^aC_{\mu\nu\rho}^b-4A^{a\lambda}C_{\mu\nu\rho\lambda}^b\right)+2\left[W_{ab}\eta_{\mu\nu\rho\lambda}^{*a}\right.\\
&\left.+(P_{ab}(W))_{[\mu\nu}B_{\rho\lambda]}^{*a}+(P_{ab}(W))_{[\mu}\eta_{\nu\rho\lambda]}^{*a}\right]C^{b\mu\nu\rho\lambda}-\left[M_{ab}^c\eta_{\mu\nu\rho\lambda}^{*a}\right.\\
&\left.+(P_{ab}^c(M))_{[\mu\nu}B_{\rho\lambda]}^{*a}+(P_{ab}^c(M))_{[\mu}\eta_{\nu\rho\lambda]}^{*a}\right]\eta^b\eta_c^{\mu\nu\rho\lambda}-\left(Q^{ab}(f)\right)_\mu\eta_{a\nu\rho\lambda}\eta_b^{\mu\nu\rho\lambda}\\
&+\tfrac{1}{4}(P_{ab}^c(M))_{\mu\nu\rho}\left(\eta^a\eta^b\eta_c^{\mu\nu\rho}-8A_\lambda^a\eta^b\eta_c^{\mu\nu\rho\lambda}\right)+(Q_{aA}(f))^\mu\eta^a\bar{\eta}_\mu^A\\
&-\left[(Q_{aA}(f))^\mu A_\mu^a+(Q_{aA}(f))^{\mu\nu}B_{\mu\nu}^{*a}-\tfrac{1}{3}(Q_{aA}(f))^{\mu\nu\rho}\eta_{\mu\nu\rho}^{*a}\right.\\
&\left.-\tfrac{1}{12}(Q_{aA}(f))^{\mu\nu\rho\lambda}\eta_{\mu\nu\rho\lambda}^{*a}\right]\bar{\eta}^A-\tfrac{1}{3!}\left[(Q_{abcd}(f))^\mu A_\mu^a+(Q_{abcd}(f))^{\mu\nu}B_{\mu\nu}^{*a}\right.\\
&\left.-\tfrac{1}{3}(Q_{abcd}(f))^{\mu\nu\rho}\eta_{\mu\nu\rho}^{*a}-\tfrac{1}{12}(Q_{abcd}(f))^{\mu\nu\rho\lambda}\eta_{\mu\nu\rho\lambda}^{*a}\right]\eta^b\eta^c\eta^d, \qquad (15)
\end{aligned}
$$

where the functions appearing in the above and denoted by $\left(Q^{ab}(f)\right)_\mu$, $(Q_{aA}(f))^\mu$, $(Q_{aA}(f))^{\mu\nu}$, $(Q_{aA}(f))^{\mu\nu\rho}$, $(Q_{aA}(f))^{\mu\nu\rho\lambda}$, $(Q_{abcd}(f))^\mu$, $(Q_{abcd}(f))^{\mu\nu}$,

$(Q_{abcd}(f))^{\mu\nu\rho}$, and $(Q_{abcd}(f))^{\mu\nu\rho\lambda}$ are withdrawn from the generic relations

$$\begin{aligned}(Q_\Lambda(f))^\mu = -f_\Lambda^A\bar{\eta}_A^{*\mu} - 2\left(P_\Lambda^A(f)\right)_\nu \bar{\eta}_A^{*\mu\nu}\\ +3\left(P_\Lambda^A(f)\right)_{\nu\rho} V_A^{*\mu\nu\rho} - \tfrac{1}{3!}\left(P_\Lambda^A(f)\right)_{\nu\rho\lambda} F_A^{\mu\nu\rho\lambda},\end{aligned} \tag{16}$$

$$(Q_\Lambda(f))^{\mu\nu} = 2f_\Lambda^A\bar{\eta}_A^{*\mu\nu} - 6\left(P_\Lambda^A(f)\right)_\rho V_A^{*\mu\nu\rho} + \tfrac{1}{2}\left(P_\Lambda^A(f)\right)_{\rho\lambda} F_A^{\mu\nu\rho\lambda}, \tag{17}$$

$$(Q_\Lambda(f))^{\mu\nu\rho} = -6f_\Lambda^A V_A^{*\mu\nu\rho} + \left(P_\Lambda^A(f)\right)_\lambda F_A^{\mu\nu\rho\lambda}, \tag{18}$$

$$(Q_\Lambda(f))^{\mu\nu\rho\lambda} = -f_\Lambda^A F_A^{\mu\nu\rho\lambda}. \tag{19}$$

The remaining P-type coefficients from (15) result from relations (13) and (14). The last three constituents of S_1 are expressed by the formulas

$$\begin{aligned}\alpha_2 = (P_{ab}(W))^{\mu\nu}\left(\eta^a C^b_{\mu\nu} - 3A^{a\rho}C^b_{\mu\nu\rho}\right) - 2\left[(P_{ab}(W))_{[\mu} B^{*a}_{\nu\rho]}\right.\\ \left.+W_{ab}\eta^{*a}_{\mu\nu\rho}\right] C^{b\mu\nu\rho} - \tfrac{1}{2}\left(P^c_{ab}(M)\right)_{\mu\nu}\left(\tfrac{1}{2}\eta^a B^{\mu\nu}_c + 3A^a_\rho\eta^{\mu\nu\rho}_c\right)\eta^b\\ +\left[\left(P^c_{ab}(M)\right)_{[\mu} B^{*a}_{\nu\rho]} + M^c_{ab}\eta^{*a}_{\mu\nu\rho}\right]\eta^b\eta^{\mu\nu\rho}_c - \tfrac{1}{2}\left[\left(P^c_{ab}(M)\right)_\mu A^{*\mu}_c\right.\\ \left.-M^c_{ab}\eta^*_c\right]\eta^a\eta^b + \left\{\left[3\left(P^c_{ab}(M)\right)_{\mu\nu} A^a_\rho + 12\left(P^c_{ab}(M)\right)_\mu B^{*a}_{\nu\rho}\right.\right.\\ \left.\left.+4M^c_{ab}\eta^{*a}_{\mu\nu\rho}\right] A^b_\lambda - 6M^c_{ab}B^{*a}_{\mu\nu}B^{*b}_{\rho\lambda}\right\}\eta^{\mu\nu\rho\lambda}_c + \tfrac{1}{2}\left(Q_{aA}(f)\right)^{\mu\nu}\left(\eta^a\bar{\eta}^A_{\mu\nu}\right.\\ \left.+A^a_{[\mu}\bar{\eta}^A_{\nu]}\right) - \left(Q_{aA}(f)\right)^{\mu\nu\rho} B^{*a}_{\mu\nu}\bar{\eta}^A_\rho - \tfrac{1}{3}\left(Q_{aA}(f)\right)^{\mu\nu\rho\lambda}\eta^{*a}_{\mu\nu\rho}\bar{\eta}^A_\lambda\\ +\left[\tfrac{1}{4}\left(Q_{abcd}(f)\right)^{\mu\nu} A^a_\mu A^b_\nu - \tfrac{1}{2}\left(Q_{abcd}(f)\right)^{\mu\nu\rho} B^{*a}_{\mu\nu}A^b_\rho\right.\\ \left.-\tfrac{1}{4!}\left(Q_{abcd}(f)\right)^{\mu\nu\rho\lambda}\left(\eta^{*a}_{[\mu\nu\rho}A^b_{\lambda]} - 2B^{*a}_{[\mu\nu}B^{*b}_{\rho\lambda]}\right)\right]\eta^c\eta^d\\ +\tfrac{1}{2}\left(Q^{ab}(f)\right)_{\mu\nu}\left(-B_{a\rho\lambda}\eta^{\mu\nu\rho\lambda}_b + \tfrac{3}{4}\eta^{\mu\alpha\beta}_a\eta^{\nu}_{b\ \alpha\beta}\right)\\ -\tfrac{1}{3}\left[\left(Q^{ab}(f)\right)_{\mu\nu\rho} A^*_{a\lambda} + \tfrac{1}{4}\left(Q^{ab}(f)\right)_{\mu\nu\rho\lambda}\eta^*_a\right]\eta^{\mu\nu\rho\lambda}_b,\end{aligned} \tag{20}$$

$$\begin{aligned}\alpha_1 = -\left(P_{ab}(W)\right)_\mu\left(\eta^a H^{b\mu} - 2A^a_\nu C^{b\mu\nu}\right) + W_{ab}\left(2B^{*a}_{\mu\nu}C^{b\mu\nu}\right.\\ \left.-\eta^a\varphi^{*b}\right) - \tfrac{1}{2}\left(P^c_{ab}(M)\right)_\mu A^a_\nu\left(2\eta^b B^{\mu\nu}_c + 3A^b_\rho\eta^{\mu\nu\rho}_c\right)\\ -M^c_{ab}\left(B^{*a}_{\mu\nu}\eta^b B^{\mu\nu}_c + A^a_\mu\eta^b A^{*\mu}_c + B^{*a}_{[\mu\nu}A^b_{\rho]}\eta^{\mu\nu\rho}_c\right)\\ -\tfrac{1}{3!}\left(Q_{aA}(f)\right)^{\mu\nu\rho}\left(A^a_{[\mu}\bar{\eta}^A_{\nu\rho]} - \eta^a V^A_{\mu\nu\rho}\right) - \tfrac{1}{2}\left(Q_{aA}(f)\right)^{\mu\nu\rho\lambda} B^{*a}_{\mu\nu}\bar{\eta}^A_{\rho\lambda}\\ -\tfrac{1}{3!}\left[\left(Q_{abcd}(f)\right)^{\mu\nu\rho} A^a_\mu A^b_\nu A^c_\rho + 3\left(Q_{abcd}(f)\right)^{\mu\nu\rho\lambda} B^{*a}_{\mu\nu}A^b_\rho A^c_\lambda\right]\eta^d\end{aligned}$$

$$+\tfrac{1}{4}\left(Q^{ab}\left(f\right)\right)_{\mu\nu\rho}\eta_{a}^{\mu\nu\sigma}B_{b\sigma}^{\rho}-\tfrac{1}{12}\left(Q^{ab}\left(f\right)\right)_{\mu\nu\rho\lambda}\eta_{a}^{\mu\nu\rho}A_{b}^{*\lambda}, \tag{21}$$

and respectively

$$\begin{aligned}\alpha_{0}=&-W_{ab}A^{a\mu}H_{\mu}^{b}+\tfrac{1}{2}M_{ab}^{c}A_{\mu}^{a}A_{\nu}^{b}B_{c}^{\mu\nu}-\tfrac{1}{4!}F_{B}^{\mu\nu\rho\lambda}\left(f_{aA}^{B}A_{[\mu}^{a}V_{\nu\rho\lambda]}^{A}+\right.\\&\left.+f_{abcd}^{B}A_{\mu}^{a}A_{\nu}^{b}A_{\rho}^{c}A_{\lambda}^{d}+\tfrac{1}{2}f^{Bab}B_{a\mu\nu}B_{b\rho\lambda}\right).\end{aligned} \tag{22}$$

In (20) and (21) the functions of the type P and Q are yielded by formulas (14) and (17)–(19). Moreover, equation (8) restricts the functions f_{aAB} to be antisymmetric in their three-form collection indices, $f_{aAB}=-f_{aBA}$. This completes the general form of the first-order deformation to the classical master equation.

The second-order deformation (the solution to equation (9)) can be shown to read as $S_{2}=\int d^{4}x\ \beta$, where

$$\beta=-\tfrac{1}{2\cdot 4!}\ H_{\mu\nu\rho\lambda}^{A}k_{AB}H^{B\mu\nu\rho\lambda}, \tag{23}$$

with

$$\begin{aligned}H_{\mu\nu\rho\lambda}^{B}=&\left[\left(P_{aA}^{B}\left(f\right)\right)_{\mu\nu\rho\lambda}\eta^{a}+\left(P_{aA}^{B}\left(f\right)\right)_{[\mu\nu\rho}A_{\lambda]}^{a}+2\left(P_{aA}^{B}\left(f\right)\right)_{[\mu\nu}B_{\rho\lambda]}^{*a}\right.\\&\left.+2\left(P_{aA}^{B}\left(f\right)\right)_{[\mu}\eta_{\nu\rho\lambda]}^{*a}+2f_{aA}^{B}\eta_{\mu\nu\rho\lambda}^{*a}\right]\bar{\eta}^{A}-\left[\left(P_{aA}^{B}\left(f\right)\right)_{[\mu\nu\rho}\bar{\eta}_{\lambda]}^{A}\right.\\&\left.-\left(P_{aA}^{B}\left(f\right)\right)_{[\mu\nu}\bar{\eta}_{\rho\lambda]}^{A}-\left(P_{aA}^{B}\left(f\right)\right)_{[\mu}V_{\nu\rho\lambda]}^{A}\right]\eta^{a}-\left[\left(P_{aA}^{B}\left(f\right)\right)_{[\mu\nu}A_{\rho}^{a}\bar{\eta}_{\lambda]}^{A}\right.\\&\left.+2\left(P_{aA}^{B}\left(f\right)\right)_{[\mu}B_{\nu\rho}^{*a}\bar{\eta}_{\lambda]}^{A}+2f_{aA}^{B}\eta_{[\mu\nu\rho}^{*a}\bar{\eta}_{\lambda]}^{A}\right]-\left(P_{aA}^{B}\left(f\right)\right)_{[\mu}A_{\nu}^{a}\bar{\eta}_{\rho\lambda]}^{A}\\&-2f_{aA}^{B}B_{[\mu\nu}^{*a}\bar{\eta}_{\rho\lambda]}^{A}+\tfrac{1}{3!}\left[\tfrac{1}{4}\left(P_{abcd}^{B}\left(f\right)\right)_{\mu\nu\rho\lambda}\eta^{a}+\left(P_{abcd}^{B}\left(f\right)\right)_{[\mu\nu\rho}A_{\lambda]}^{a}\right.\\&\left.+2\left(P_{abcd}^{B}\left(f\right)\right)_{[\mu\nu}B_{\rho\lambda]}^{*a}+2\left(P_{abcd}^{B}\left(f\right)\right)_{[\mu}\eta_{\nu\rho\lambda]}^{*a}+2f_{abcd}^{B}\eta_{\mu\nu\rho\lambda}^{*a}\right]\eta^{b}\eta^{c}\eta^{d}\\&-\tfrac{1}{2}\left[\left(P_{abcd}^{B}\left(f\right)\right)_{[\mu\nu}A_{\rho}^{a}A_{\lambda]}^{b}+2\left(P_{abcd}^{B}\left(f\right)\right)_{[\mu}B_{\nu\rho}^{*a}A_{\lambda]}^{b}\right.\\&\left.+2f_{abcd}^{B}\left(\eta_{[\mu\nu\rho}^{*a}A_{\lambda]}^{b}-2B_{[\mu\nu}^{*a}B_{\rho\lambda]}^{*b}\right)\right]\eta^{c}\eta^{d}-\left[\left(P_{abcd}^{B}\left(f\right)\right)_{[\mu}A_{\nu}^{a}A_{\rho}^{b}A_{\lambda]}^{c}\right.\\&\left.+2f_{abcd}^{B}B_{[\mu\nu}^{*a}A_{\rho}^{b}A_{\lambda]}^{c}\right]\eta^{d}+\tfrac{1}{2}\left(P^{Bab}\left(f\right)\right)_{\mu\nu\rho\lambda}\eta_{a\alpha\beta\gamma\delta}\eta_{b}^{\alpha\beta\gamma\delta}\\&-\left(P^{Bab}\left(f\right)\right)^{\alpha\beta\gamma}\eta_{a\alpha\beta\gamma}\eta_{b\mu\nu\rho\lambda}+\tfrac{3}{8}\left(P^{Bab}\left(f\right)\right)^{\alpha\beta}\eta_{a\alpha\beta[\mu}\eta_{\nu\rho\lambda]b}\\&+\left[\left(P^{Bab}\left(f\right)\right)^{\alpha\beta}B_{a\alpha\beta}+2\left(P^{Bab}\left(f\right)\right)^{\alpha}A_{a\alpha}^{*}-2f^{Bab}\eta_{a}^{*}\right]\eta_{b\mu\nu\rho\lambda}\\&+\tfrac{1}{2}\left(P^{Bab}\left(f\right)\right)^{\sigma}B_{a\sigma[\mu}\eta_{\nu\rho\lambda]b}-\tfrac{1}{2}f^{Bab}A_{a[\mu}^{*}\eta_{\nu\rho\lambda]b}+f_{aA}^{B}A_{[\mu}^{a}V_{\nu\rho\lambda]}^{A}\end{aligned}$$

$$+f^{B}_{abcd}A^a_\mu A^b_\nu A^c_\rho A^d_\lambda + \tfrac{1}{3!}f^{Bab}B_{a[\mu\nu}B_{\rho\lambda]b}. \tag{24}$$

In the meantime, equation (9) requests that the various functions depending on the undifferentiated scalar fields that parameterize the first-order deformation are subject to the equations

$$W_{ea}\frac{\partial W_{bc}}{\partial\varphi_e} + W_{eb}\frac{\partial W_{ca}}{\partial\varphi_e} + W_{ec}M^e_{ab} = 0, \quad W_{e[a}\frac{\partial M^d_{bc]}}{\partial\varphi_e} + M^d_{e[a}M^e_{bc]} = 0, \tag{25}$$

$$-W_{e[a}\frac{\partial f^A_{b]B}}{\partial\varphi_e} - M^e_{ab}f^A_{eB} + f^A_{aE}f^E_{bB} - f^A_{bE}f^E_{aB} = 0, \quad f^{Aae}W_{eb} = 0, \tag{26}$$

$$W_{f[a}\frac{\partial f^A_{bcde]}}{\partial\varphi_f} + f^A_{f[abc}M^f_{de]} - f^E_{[abcd}f^A_{e]E} = 0, \quad f^{(A}_{eC}f^{B)ae} = 0, \tag{27}$$

$$W_{ec}\frac{\partial f^{Aab}}{\partial\varphi_e} + f^{Ae(a}M^{b)}_{ec} - f^A_{cM}f^{Mab} = 0, \quad f^{(A}_{ebcd}f^{B)ae} = 0. \tag{28}$$

Further, by direct computation we infer that $(S_1, S_2) = 0$, so all the other deformations, of order three or higher, can be taken to vanish, $S_3 = S_4 = \cdots = S_k = \cdots = 0$. In conclusion, the full deformed solution to the master equation for the model under study, which is consistent to all orders in the coupling constants, can be written as $\bar{S} = S + \lambda\int d^4x\,(\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4) + \lambda^2\int d^4x\ \beta$, where its first-order components are listed in (10), (15), (20)–(22) and β is expressed by (23). From the deformed solution to the master equation we can extract all the information on the (gauge) structure of the resulting interacting model.

Thus, the piece of antighost number zero from the deformed solution $\bar{S}$ is precisely the Lagrangian action of the coupled model and has the expression

$$\hat{S}[A^a_\mu, H^a_\mu, \varphi_a, B^{\mu\nu}_a, V^A_{\mu\nu\rho}] = \int d^4x\left(H^a_\mu D^\mu\varphi_a + \tfrac{1}{2}B^{\mu\nu}_a\bar{F}^a_{\mu\nu} - \tfrac{1}{2\cdot 4!}\bar{F}^A_{\mu\nu\rho\lambda}\bar{F}^{\mu\nu\rho\lambda}_A\right), \tag{29}$$

where we employed the notations

$$D_\mu\varphi_a = \partial_\mu\varphi_a + \lambda W_{ab}A^b_\mu, \quad \bar{F}^a_{\mu\nu} = \partial_{[\mu}A^a_{\nu]} + \lambda M^a_{bc}A^b_\mu A^c_\nu, \tag{30}$$

$$\begin{aligned}\bar{F}^A_{\mu\nu\rho\lambda} \;=\; & F^A_{\mu\nu\rho\lambda} + \lambda\left(f^A_{abcd}A^a_\mu A^b_\nu A^c_\rho A^d_\lambda + f^A_{aB}A^a_{[\mu}V^B_{\nu\rho\lambda]}\right.\\ & \left.+\tfrac{1}{3!}f^{Aab}B_{a[\mu\nu}B_{\rho\lambda]b}\right).\end{aligned} \tag{31}$$

Under the general hypotheses mentioned at the beginning of this paper, formula (29) gives the most general form of the action describing the four-dimensional interactions between a collection of BF models and a set of three-form gauge fields, whose free limit is (1). The action (29) is invariant under the deformed gauge transformations

$$\bar{\delta}_\epsilon \varphi_a = -\lambda W_{ab}\epsilon^b, \quad \bar{\delta}_\epsilon A^a_\mu = (D_\mu)^a{}_b \epsilon^b + \tfrac{\lambda}{12} f^{ab}_A \bar{F}^A_{\mu\nu\rho\lambda}\epsilon^{\nu\rho\lambda}_b, \tag{32}$$

$$\begin{aligned} &\bar{\delta}_\epsilon B^{\mu\nu}_a = -3 (D_\rho)_a{}^b \epsilon^{\mu\nu\rho}_b + \lambda \left(2W_{ab}\epsilon^{b\mu\nu} - M^c_{ab}B^{\mu\nu}_c\epsilon^b\right) \\ &+\tfrac{\lambda}{2}\bar{F}^{\mu\nu\rho\lambda}_A \left(f^A_{abcd}A^b_\rho A^c_\lambda \epsilon^d + f^A_{aB}\epsilon^B_{\rho\lambda}\right), \end{aligned} \tag{33}$$

$$\begin{aligned} &\bar{\delta}_\epsilon H^a_\mu = 2\left(\tilde{D}^\nu\right)^a{}_b \epsilon^b_{\mu\nu} - \tfrac{3\lambda}{2}\frac{\partial M^d_{bc}}{\partial\varphi_a}A^{b\nu}A^{c\rho}\epsilon_{d\mu\nu\rho} - \tfrac{\lambda}{12}\frac{\partial f^{Abc}}{\partial\varphi_a}B_{b\mu\alpha}\epsilon_{c\beta\gamma\delta}\bar{F}^{\alpha\beta\gamma\delta}_A \\ &-\tfrac{\lambda}{2}\bar{F}^A_{\mu\nu\rho\lambda}\left(\frac{\partial f^B_{bA}}{\partial\varphi_a}A^{b\nu}\epsilon^{\rho\lambda}_B - \tfrac{1}{3}\frac{\partial f^B_{bA}}{\partial\varphi_a}\epsilon^b V^{\mu\nu\rho}_B - \tfrac{1}{3}\frac{\partial f_{Abcde}}{\partial\varphi_a}A^{b\nu}A^{c\rho}A^{d\lambda}\epsilon^e\right) \\ &-\lambda\left(\frac{\partial W_{bc}}{\partial\varphi_a}H^c_\mu - \frac{\partial M^d_{bc}}{\partial\varphi_a}A^{c\nu}B_{d\mu\nu}\right)\epsilon^b, \end{aligned} \tag{34}$$

$$\bar{\delta}_\epsilon V^A_{\mu\nu\rho} = \left(D_{[\mu}\right)^A{}_B \epsilon^B_{\nu\rho]} - \lambda f^A_{aB}V^B_{\mu\nu\rho}\epsilon^a + \lambda f^A_{abcd}A^a_\mu A^b_\nu A^c_\rho \epsilon^d + \tfrac{\lambda}{2} f^{Aab}\sigma^{\alpha\beta}B_{a\alpha[\mu}\epsilon_{\nu\rho]\beta b}, \tag{35}$$

where we used the notations

$$(D_\mu)^a{}_b = \delta^a_b\partial_\mu - \lambda M^a_{bc}A^c_\mu, \quad (D_\mu)_a{}^b = \delta^b_a\partial_\mu + \lambda M^b_{ac}A^c_\mu, \tag{36}$$

$$\left(\tilde{D}_\mu\right)^a{}_b = \delta^a_b\partial_\mu - \lambda\frac{\partial W_{bc}}{\partial\varphi_a}A^c_\mu, \quad (D_\mu)^A{}_B = \delta^A_B\partial_\mu + \lambda f^A_{aB}A^a_\mu. \tag{37}$$

The gauge transformations (32)–(35) remain second-order reducible, but the reducibility relations only hold on-shell (where on-shell means here on the stationary surface of the field equations for the action (29)). These relations have an intricate, but not illuminating form, and therefore we will skip them. The gauge algebra accompanying the deformed gauge transformations (32)–(35) is open, in contrast to the original one, which is Abelian.

At this point, we have the entire information on the gauge structure of the deformed theory. From (29)–(31) we observe that there appear two main types of vertices. The first type, $\lambda\left(H^a_\mu W_{ab}A^{b\mu} + \frac{1}{2}B^{\mu\nu}_b M^b_{ac}A^a_\mu A^c_\nu\right)$, describes the self-interactions among the BF fields in the absence of the three-forms

and has been previously obtained in the literature [30, 31]. The second kind of vertices can be put in the form

$$
\begin{aligned}
&-\tfrac{\lambda}{4!}k_{AB}\partial^{[\mu}V^{\nu\rho\lambda]A}\left(f^{B}_{abcd}A^{a}_{\mu}A^{b}_{\nu}A^{c}_{\rho}A^{d}_{\lambda}+f^{B}_{aC}A^{a}_{[\mu}V^{C}_{\nu\rho\lambda]}+\tfrac{1}{3!}f^{Bab}B_{a[\mu\nu}B_{\rho\lambda]b}\right)\\
&-\tfrac{\lambda^{2}}{2\cdot 4!}k_{AB}\left(f^{A}_{abcd}A^{a}_{\mu}A^{b}_{\nu}A^{c}_{\rho}A^{d}_{\lambda}+f^{A}_{aC}A^{a}_{[\mu}V^{C}_{\nu\rho\lambda]}+\tfrac{1}{3!}f^{Aab}B_{a[\mu\nu}B_{\rho\lambda]b}\right)\\
&\times\left(f^{B}_{emnp}A^{e\mu}A^{m\nu}A^{n\rho}A^{p\lambda}+f^{B}_{eD}A^{e[\mu}V^{\nu\rho\lambda]D}+\tfrac{1}{3!}f^{Bmn}B^{[\mu\nu}_{m}B^{\rho\lambda]}_{n}\right). \qquad (38)
\end{aligned}
$$

We remark that (38) contains some vertices involving only the BF fields

$$
\begin{aligned}
&-\tfrac{\lambda^{2}}{2\cdot 4!}k_{AB}\left(f^{A}_{abcd}A^{a}_{\mu}A^{b}_{\nu}A^{c}_{\rho}A^{d}_{\lambda}+\tfrac{1}{3!}f^{Aab}B_{a[\mu\nu}B_{\rho\lambda]b}\right)\\
&\times\left(f^{B}_{emnp}A^{e\mu}A^{m\nu}A^{n\rho}A^{p\lambda}+\tfrac{1}{3!}f^{Bmn}B^{[\mu\nu}_{m}B^{\rho\lambda]}_{n}\right), \qquad (39)
\end{aligned}
$$

whose existence is nevertheless induced by the presence of the three-form gauge fields. Indeed, in the absence of these fields ($k_{AB}=0$) (39) vanishes. The remaining terms from (38) produce cross-couplings between the BF fields and the three-forms. From (38) it is clear that the one-forms H^{a}_{μ} (from the BF sector) cannot be coupled to the three-form gauge fields. The deformed gauge transformations (32)–(35) exhibit a rich structure, which includes, among others, the generalized covariant derivatives (36) and (37). It is interesting to notice that the presence of the three-forms modifies the gauge transformations of A^{a}_{μ}, $B^{\mu\nu}_{a}$, and H^{a}_{μ} by terms proportional with the deformed field strength $\bar{F}^{A}_{\mu\nu\rho\lambda}$. Although the one-forms H^{a}_{μ} do not couple to the three-form gauge fields, their gauge transformations contain the gauge parameters $\epsilon^{\rho\lambda}_{B}$, specific to the three-form sector. Similarly, the BF sector contributes to the gauge transformations of the three-forms.

The previous results have been obtained in $D=4$ space-time dimensions. We mention that the resulting cross-coupling terms originate in the pieces from (10) proportional with $Q_{aA}(f)$, $Q_{abcd}(f)$, and $Q^{ab}(f)$. These pieces are consistent independently one from another (and also from the other terms present in (10)) at the level of the first-order deformation. Let us consider now the case $D>4$ (for $D<4$ the field strengths of the three-forms vanish, such that no cross-couplings occur). In this case the gauge transformations (2)–(3) from the BF sector are $(D-2)$-order reducible. This implies the introduction of a larger spectrum of ghosts and antifields for the BF sector than in $D=4$. In addition, the first-order deformation will accordingly stop at antighost number D, $S_1=\int d^{D}x(\alpha_0+\cdots+\alpha_D)$. Standard cohomological arguments can be used in order to establish that α_D will depend only on

the BRST generators from the BF sector. As a consequence, all the components from the first-order deformation generated by α_D will contribute only to pure BF couplings. On the other hand, basic cohomological arguments ensure that the three-form BRST sector will occur non-trivially in the first-order deformation only starting with terms of antighost number four (just like in $D = 4$) via the pieces from α_4 proportional with $Q_{aA}(f)$ and $Q_{abcd}(f)$ (see (10); the piece proportional with $Q^{ab}(f)$ is absent in $D > 4$ due to the fact that the ghosts $\eta_{a\mu\nu\rho\lambda}$ are no longer γ-invariant). Just like in $D = 4$, the terms from α_4 proportional to $Q_{aA}(f)$ and $Q_{abcd}(f)$ will be consistent independently one from each other (and also from other pure BF terms) and will yield the same results like in the case $D = 4$. By contrast, all the contributions coming from the term proportional with $Q^{ab}(f)$ must be discarded from the first- and also from the second-order deformations in $D > 4$ (in particular, the term $\frac{1}{3!}f^{Aab}B_a^{[\mu\nu}B_b^{\rho\lambda]}$ is absent from the deformed action and accompanying gauge transformations). In conclusion, the interacting action in $D > 4$ will have a form similar to (29) up to the fact that $\bar{F}^{A\mu\nu\rho\lambda}$ will lack the term $\frac{1}{3!}f^{Aab}B_a^{[\mu\nu}B_b^{\rho\lambda]}$. In this situation the deformed gauge transformations of A_μ^a, H_μ^a, and $V_{\mu\nu\rho}^A$ will no longer contain terms proportional with the gauge parameters $\epsilon_a^{\mu\nu\rho}$. It is understood that the deformed field strength $\bar{F}_{\mu\nu\rho\lambda}^A$ must be replaced everywhere with its new expression, as explained in the above. The previous discussion emphasizes that the case $D = 4$ is a privileged situation because it outputs the richest gauge structure for the cross-couplings between the BF models and the three-forms.

Our procedure is consistent provided the equations (25)–(28) are shown to possess solutions. In the sequel we give some classes of solutions to these equations, without pretending to exhaust all their possible solutions.

A first class of solutions is given by $M_{ab}^c = \frac{\partial W_{ab}}{\partial \varphi_c}$, $f_{aB}^A = k^m \lambda_B^A W_{am}$, $f_{abcd}^A = \mu^A f_{e[ab}\frac{\partial W_{cd]}}{\partial \varphi_e}$, $f^{Aab} = 0$, where k^m are some arbitrary constants and μ^A together with λ_B^A are some constants subject to the conditions $\lambda_B^A \mu^B = 0$, while the non-degenerate matrix of elements W_{ab} must satisfy the equations

$$W_{e[a}\frac{\partial W_{bc]}}{\partial \varphi_e} = 0. \tag{40}$$

We remark that all the non-vanishing solutions are in this case parameterized by the antisymmetric functions W_{ab}.

We briefly review the basic notions on Poisson manifolds. If P denotes an arbitrary Poisson manifold, then this is equipped with a Poisson bracket

$\{,\}$ that is bilinear, antisymmetric, subject to a Leibnitz-like rule, and satisfies a Jacobi-type identity. If $\{X^i\}$ are some local coordinates on P, then there exists a two-tensor $\mathcal{P}^{ij} \equiv \{X^i, X^j\}$ (the Poisson tensor) that uniquely determines the Poisson structure together with the Leibnitz rule. This two-tensor is antisymmetric and transforms in a covariant manner under coordinate transformations. Jacobi's identity for the Poisson bracket $\{,\}$ expressed in terms of the Poisson tensor reads as $\mathcal{P}^{ij}_{,k}\mathcal{P}^{kl} + \text{cyclic}\,(i,j,l) = 0$, where $\mathcal{P}^{ij}_{,k} \equiv \partial\mathcal{P}^{ij}/\partial X^k$. In view of this discussion we can interpret the functions W_{ab} like the components of a two-tensor on a Poisson manifold with the target space locally parameterized by the scalar fields φ_e.

Another class of solutions to (25)–(28) can be expressed as $W_{ab} = 0$, $f^A_{aB} = 0$, $f^A_{abcd} = 0$, $f^{Aab} = \mu^{ab}\xi^A\hat{M}(\varphi)$, $M^c_{ab} = C^c{}_{ab}M(\varphi)$, with $\hat{M}$ and M arbitrary functions of the undifferentiated scalar fields, ξ^A some arbitrary constants, and μ^{ab} the inverse of the Killing metric of a semi-simple Lie algebra with the structure constants $C^c{}_{ab}$, where, in addition $C_{abc} = \bar{\mu}_{ad}C^d{}_{ab}$ (with $\bar{\mu}_{ad}\mu^{de} = \delta^e_a$) must be completely antisymmetric.

A third class of solutions can be written as $W_{ab} = 0$, $f^A_{aB} = 0$, $f^{Aab} = 0$, $M^c_{ab} = \bar{C}^c{}_{ab}\hat{N}(\varphi)$, $f^A_{abcd} = \bar{\xi}^A\bar{f}_{e[ab}\bar{C}^e{}_{cd]}N(\varphi)$, where $\hat{N}$ and N are some arbitrary functions of the undifferentiated scalar fields, $\bar{\xi}^A$ and $\bar{f}_{eab}$ denote some arbitrary constants, and $\bar{C}^c{}_{ab}$ are the structure constants of a (in general not semi-simple) Lie algebra. Let us particularize the last solutions to the case where $\bar{C}^c{}_{ab} = \bar{k}^c\bar{W}_{ab}$, $\hat{N}(\varphi) = N(\varphi) = \frac{d\hat{w}\left(\bar{k}^m\varphi_m\right)}{d\left(\bar{k}^n\varphi_n\right)}$, with $\bar{k}^c$ some arbitrary constants, $\hat{w}$ an arbitrary, smooth function depending on $\bar{k}^m\varphi_m$, and $\bar{W}_{ab}$ some constants satisfying the relations $\bar{W}_{a[b}\bar{W}_{cd]} = 0$. Obviously, the last relations ensure the Jacobi identity for the structure constants $\bar{C}^c{}_{ab}$. Replacing back the particular form of $\bar{C}^c{}_{ab}$, $\hat{N}$, and N into the initial solutions from the third class, we find $W_{ab} = 0$, $f^A_{aB} = 0 = f^{Aab}$, $M^c_{ab} = \frac{\partial\hat{W}_{ab}}{\partial\varphi_c}$, and $f^A_{abcd} = \bar{\xi}^A\bar{f}_{e[ab}\frac{\partial\hat{W}_{cd]}}{\partial\varphi_e}$, where $\hat{W}_{ab} = \bar{W}_{ab}\frac{d\hat{w}\left(\bar{k}^m\varphi_m\right)}{d\left(\bar{k}^n\varphi_n\right)}$. It is easy to see, due to $\bar{W}_{a[b}\bar{W}_{cd]} = 0$, that $\hat{W}_{ab}$ satisfy the Jacobi identity for a Poisson manifold, $\hat{W}_{e[a}\frac{\partial\hat{W}_{bc]}}{\partial\varphi_e} = 0$. The above discussion emphasizes that we can generate solutions correlated with a Poisson manifold even if $W_{ab} = 0$. In this situation the Poisson two-tensor results from a Lie algebra. It is interesting to remark that the same equations, namely $\bar{W}_{a[b}\bar{W}_{cd]} = 0$, ensure the Jacobi identities for both the Lie algebra and the corresponding Poisson manifold. These equations possess at least two types of solutions: $\bar{W}_{ab} = \varepsilon_{ijk}e^i_a e^j_b e^k_c\rho^c$, with i,j,$k = \overline{1,3}$, and respectively

$\bar{W}_{ab} = \varepsilon_{\bar{a}\bar{b}\bar{c}} l_a^{\bar{a}} l_b^{\bar{b}} l_c^{\bar{c}} \bar{\rho}^c$, with $\bar{a}$,$\bar{b}$,$\bar{c} = \overline{1,4}$, where e_a^i, ρ^c, $l_a^{\bar{a}}$, and $\bar{\rho}^c$ are all constants and ε_{ijk} together with $\varepsilon_{\bar{a}\bar{b}\bar{c}}$ are completely antisymmetric symbols, defined via the conventions $\varepsilon_{123} = \varepsilon_{124} = \varepsilon_{134} = \varepsilon_{234} = +1$.

Finally, we consider the case where there exist some independent constants $\hat{k}^a$ such that $\hat{k}^a W_{ab} = 0$, so W_{ab} is degenerate. In this situation a class of solutions to (25)–(28) is expressed by $M_{bc}^a = \frac{\partial W_{bc}}{\partial \varphi_a}$, $f_{aB}^A = k^m \lambda_B^A W_{am}$, $f_{abcd}^A = 0$, and $f^{Aab} = \hat{k}^a \hat{k}^b \bar{M}(\hat{u})$, where $\bar{M}$ is an arbitrary function of $\hat{u} = \hat{k}^a \varphi_a$, W_{ab} must satisfy the Jacobi identity (40), k^m are some arbitrary constants, and μ^A are null vectors for λ_B^A. In this case W_{ab} depend on the undifferentiated scalar fields, but not necessarily through $\hat{u}$. The solution f^{Aab} is also degenerate since it possesses the null vectors $\varepsilon_{bc}(\varphi_e)\hat{k}^c$, with ε_{bc} some antisymmetric, arbitrary functions of the undifferentiated scalar fields.

In each of the four cases studied in the above the entire gauge structure of the interacting model can be obtained by substituting the corresponding solution into the formulas (29)–(37). We remark that in all these four cases we obtain interaction vertices among the BF fields induced by the presence of the three-form gauge fields as well as vertices that describe cross-couplings between BF fields and three-forms.

To conclude with, in this paper we have investigated the consistent interactions that can be introduced between a collection of BF theories and a set of three-form gauge fields. Starting with the BRST differential for the free theory, we give the consistent first-order deformation of the solution to the master equation, and obtain that it is parameterized by several kinds of functions depending only on the undifferentiated scalar fields. Next, we determine the second-order deformation, whose existence imposes certain restrictions with respect to these types of functions. Based on these restrictions, we show that we can take all the remaining higher-order deformations to vanish. As a consequence of our procedure, we are led to an interacting gauge theory with deformed gauge transformations, a non-Abelian gauge algebra that only closes on-shell, and on-shell, second-order reducibility relations. Finally, we investigate the equations that restrict the functions parameterizing the deformed solution to the master equation, and give some particular classes of solutions.

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